

Plane Algebraic Curves – Problem Set 4

due Tuesday, June 9 at 10:15

- (1) Show by example that Max Noether's Theorem as in Corollary 4.12 is in general false ...
 - (a) if the ground field is not algebraically closed; or
 - (b) if the curve F is not assumed to be smooth.
- (2) Prove the following converse of Pascal's Theorem:
Let $P_1, \dots, P_6 \in \mathbb{P}^2$ be distinct points so that the six lines $\overline{P_1P_2}, \overline{P_2P_3}, \dots, \overline{P_5P_6}, \overline{P_6P_1}$ (which can be thought of as the sides of the hexagon with vertices $P_1, \dots, P_6$) are also distinct. Let $P = \overline{P_1P_2} \cap \overline{P_4P_5}$, $Q = \overline{P_2P_3} \cap \overline{P_5P_6}$, $R = \overline{P_3P_4} \cap \overline{P_6P_1}$ be the intersection points of opposite sides of the hexagon. If P, Q, R lie on a line, then $P_1, \dots, P_6$ lie on a conic.
- (3)
 - (a) Show that a (not necessarily irreducible) reduced curve of degree d in $\mathbb{P}^2$ has at most $\binom{d}{2}$ singular points.
 - (b) Find an example for each d in which this maximal number of singular points is actually reached.
- (4) Use Max Noether's Theorem to show the following statements about smooth projective cubics F and G over an algebraically closed field:
 - (a) (*Cayley-Bacharach Theorem*) Assume that F and G intersect in exactly 9 points $P_1, \dots, P_9$. Moreover, let E be another cubic that also contains the first eight points $P_1, \dots, P_8$. Prove that E then also contains P_9 .
 - (b) Any line through two inflection points of F as in Exercise 2 of Problem Set 3 passes through a third inflection point of F .